

PERIPHERAL TEMPERATURE VARIATION IN THE WALL OF A NONCIRCULAR DUCT—AN EXPERIMENTAL INVESTIGATION

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Abstract—This paper is concerned with the peripheral variation of the temperature in the wall of a straight noncircular duct, with special reference to the prediction of the temperature in the cladding of the fuel rods of a pressurized water reactor, in the event of a loss of coolant. A simple model of the conjugate heat transfers in the wall and the fluid is used to predict the temperature variation around the wall. To test the theory, experiments have been made to determine wall temperatures in a cusped duct using air as the working fluid for a range of fluid flow rates. Overall pressure drop and heat transfer measurements for friction factor and average heat transfer coefficient indicate the inadequacy of the hydraulic radius concept in the case of the very noncircular geometry used in the experiments, and the effect of asymmetry in heat transfer. It is thought that these heat transfer experiments are the first for this particular geometry.

NOMENCLATURE

A	cross sectional area of duct
D_e	equivalent diameter of duct, $4A/P$
f	friction factor, $2\tau_w/\rho u^2$
Gr	Grashof number
$h, (\bar{h})$	convective heat transfer coefficient (average value)
k	thermal conductivity of wall material
L	peripheral length of element of heat transfer surface
n	exponent in equation (A1)
Nu	average Nusselt number
P	perimeter of duct
q_w	surface heat flux
Re	Reynolds number
T_w, T_b	wall temperature, bulk temperature of fluid
t	thickness of wall (cladding)
\bar{u}	average velocity
x	peripheral distance measured from corner of duct
z	axial distance.

Greek symbols

θ	temperature difference, $T_w - T_b$
ρ	fluid density
τ_w	average wall shear stress.

INTRODUCTION

FULLY developed flow and heat transfer in a straight noncircular duct of constant cross sectional area have important practical applications, and numerous examples in the engineering field are readily called to mind. Distinctive features of this flow and heat transfer situation, are the variation of the heat transfer coefficient and in general, the temperature around the wall of the duct. In many cases, the laminar flow problem is amenable to analytical solution and the friction factor and Nusselt number for a wide range of

simple geometries and idealized boundary conditions are listed for example by Holman [1]. The turbulent flow case is considerably more complex necessitating modelling of the turbulence and turbulence-driven secondary flows which exist in the plane of the duct cross section. The turbulent flow problem also requires considerable numerical effort, examples of studies of this problem being those by Launder and Ying [2], and Aly *et al.* [3], who dealt with square and triangular geometries, respectively. These solutions again refer to cases with relatively simple thermal boundary conditions and do not cater for the possibly large variation of temperature around the wall of the duct in the real case. This peripheral or azimuthal temperature variation is a consequence of the conjugate wall heat conduction which, in general, must be solved simultaneously with the convection problem. Large peripheral temperature gradients in the wall of a duct may result in prohibitively large local temperatures or 'hot spots' so that their prediction is very important particularly in the case of the highly rated heat transfer surface.

The present study has its origin in an extensive investigation of flow and heat transfer in a very noncircular cusped duct. In the event of the loss of coolant in a pressurized water reactor, it has been suggested that local ballooning or swelling of the cladding of adjacent fuel elements may result in contact of the cladding. This may lead to the production of 'single-connected' channels which have a shape resembling an astroid. The case of turbulent flow and heat transfer in such a four-cusp duct (which is shown diagrammatically in Fig. A1) has been studied by Haque *et al.* [4], who focused attention on the development of the numerical method and the modelling of the turbulence characteristics and secondary flows. By assuming a wall temperature which is constant in the peripheral direction, the conjugate wall heat conduction and fluid convection problem was circumvented. It is now recognized that there is a need for the prediction of the all important

peripheral wall temperature variation in the more realistic situation, in which case, the heat conduction problem must be included in the analysis. How this might be effected without recourse to the extensive numerical effort necessary for the solution of the conjugate heat transfer problem in the cusped duct, is one of the subjects of this paper. A relatively simple analysis is developed treating the wall element in the same manner as a fin. To simulate the pressurized water reactor cladding problem, uniform heat flux is imposed along the length of the 'fin' to represent the fuel energy release, while a variable heat transfer coefficient along the length represents the variable convection around the blocked reactor channel. A brief outline of the model and the associated analysis is given in the Appendix, while a fuller account is available in a companion paper [5], dealing with various amounts of blockage and the dependence of peripheral wall temperature variation on wall thickness, thermal conductivity and the coolant flow rate.

The major part of this paper refers to a simple experimental programme of work, the results of which may be used to critically examine the simple theoretical analysis mentioned previously. The experimental work was concerned with heat transfer in a two-cusp duct for which the problems are similar to those of the four-cusp geometry. This experimental work, its results, and how it relates to the reactor cladding problem described earlier, is outlined in the following sections. A further purpose of the experimental investigation was the correlation of the overall friction factor and the average heat transfer coefficient in ducts with very noncircular cross sections and the role of the equivalent diameter in this connection. For completeness, therefore, some of these results are included in the section dealing with the experimental data.

APPARATUS AND EXPERIMENTAL PROCEDURE

The idealized geometry for the simulation of flow and heat transfer in a blocked channel of a pressurized water reactor is a straight four-cusp duct as explained previously. However, for the purpose of testing the simple numerical prediction method for determining peripheral variations in wall temperature in such a geometry, the shape (two cusp) shown in Fig. 1 suffices, and this was chosen for the experimental investigation.

The experimental duct, which was 1.83 m long, consisted of a curved 0.16 mm thick stainless steel sheet and timber assembly as shown in Fig. 1. Local variations in the thickness of the stainless steel plate were found to be very small, permitting the assumption of a uniform heat generation condition throughout the heated boundary of the duct. (A transparent Perspex sheet replaced the steel sheet for pressure drop measurements.) The equivalent diameter of the two-cusp duct was 13.2 mm. While water as a coolant is of interest in pressurized water reactor applications, air was chosen as the working fluid in the present study.

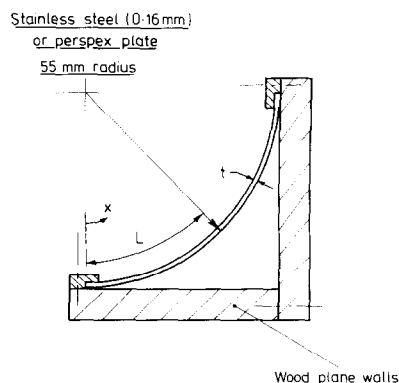


FIG. 1. Experimental two-cusp duct.

Therefore, the ancillary equipment included two fans, a settling chamber and an airflow meter.

For completeness, the experimental work included pressure drop measurements in isothermal flow and for this purpose the curved Perspex sheet accommodated static pressure tappings along its length. (The smaller capacity fan was used specifically for the pressure drop measurements.) Heating of the stainless steel plate in the heat transfer tests was effected using a 10 kVA single phase transformer, connection being made via copper bus bars brazed to the ends of the plate. This method of heating provided uniform heat generation in the plate to simulate the fuel-cladding interfacial heat flux in the real situation. The power input was measured by determining the voltage and current (through a current transformer) using a Schlumberger Digital Multimeter. The local temperatures of the plate were obtained with either chromel–alumel or copper–constantan thermocouples, which were either of the patch type or junctions soldered to the plate's outermost surface, in conjunction with a Comark Electrical Thermometer. The thermocouples were attached on the plate centreline along the length, and at two stations 1.23 and 1.67 m downstream from entry, thermocouples were located in the peripheral or azimuthal direction to determine the important temperature variation around the plate. (The angular location of the thermocouple junctions relative to the corner of the duct, are as shown in the figures relating to the experimental temperature data.) This system of thermocouples served to check the heat transfer development. For fully established flow and heat transfer with uniform total energy generation in the plate, the wall temperature is given by

$$T_w = \text{function}(\theta) + \text{const. } z. \quad (1)$$

That is, the axial temperature gradient, $(\partial T_w / \partial z)$, is a constant for all angular positions. The attachment of thermocouples on the outer surface of the plate is compatible with the model described in the Appendix. The temperature drop across the plate thickness is negligibly small in any event.

The stainless steel plate was carefully insulated with fibreglass matting to minimize heat losses. In order to assess the magnitude of the heat loss, the apparatus was

operated at low power with no coolant flow and the wall temperature was measured. These data were then used to correct for the power input in normal operation. The corrected power input was employed in the evaluation of the bulk temperature gradient, which in the developed region must be the same as that for the wall in accordance with equation (1).

All the tests were conducted under steady-state conditions, the electrical input power being chosen to match the flow rate which was measured by the airflow meter on the fan suction. Data reduction followed the normal pattern of the evaluation of Reynolds number, friction factor in the preliminary tests, and finally fully developed average heat transfer coefficient (i.e. heat transfer coefficient based on average surface heat flux and average wall temperature to bulk temperature difference). This 'average' heat transfer coefficient was employed in the prediction of wall temperature variation using the model described in the Appendix.

The effect of free convection within the flow was estimated as being negligible, since the appropriate ratio (Gr/Re^2) was found to be very much less than unity even for the smallest flow rate.

The present experimental heat transfer situation is clearly one of asymmetrical heating since only the curved boundary (i.e. stainless steel plate), transfers energy to the flow. Provided the average experimental heat transfer coefficient for this condition is used when making a comparison with the corresponding experimental temperature data, the nature of the thermal boundary condition is irrelevant. Of course differences between the measured average heat transfer coefficient and that predicted by the use of the equivalent diameter in pipe flow correlations do exist. This difference is a result of both the effects of asymmetry of heat transfer and the inadequacy of the hydraulic radius concept itself in the case of very noncircular ducts. If the average heat transfer coefficient to be used in the theoretical prediction (see Appendix) is to be taken from an existing correlation, then of course that correlation must refer to the particular geometry and thermal boundary conditions.

In the following section, a selection of experimental data is presented. Firstly the experimental wall temperatures are considered along with a prediction using the experimentally measured average heat transfer coefficients. These last coefficients are also examined in relation to those obtainable using the equivalent diameter in pipe flow correlations.

RESULTS AND DISCUSSION

While the investigation was primarily concerned with heat transfer, preliminary pressure drop measurements were made to examine the hydrodynamic flow development along the length of the duct. The corresponding fully developed friction factors are shown plotted in the conventional manner in Fig. 2 together with the Blasius equation and the experi-

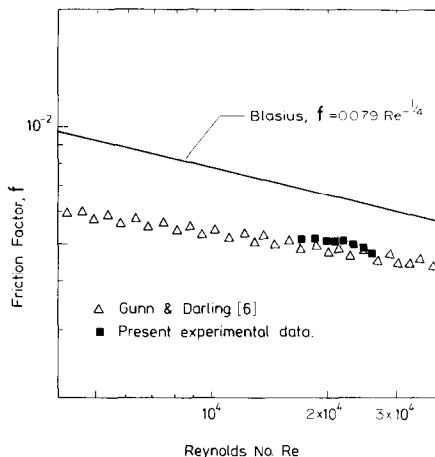


FIG. 2. Friction factor.

mental results of Gunn and Darling [6]. Very good agreement with the data of ref. [6] was found confirming the inadequacy of the hydraulic radius concept in the case of the very noncircular geometry of the two-cusp duct when comparison is made with the Blasius correlation.

The fully developed nature of the heat transfer conditions at two stations 93.2 and 126.5 equivalent diameters downstream is clearly shown in Fig. 3 where the peripheral wall temperature profiles exhibit identical shapes. This is consistent with equation (1) which describes the fully developed temperature field for the specified thermal boundary conditions. Terminal heat transfer conditions are also confirmed by plotting axial wall temperature profiles at various azimuthal positions and this has been done in Fig. 4. Clearly, $(\partial T_w / \partial z)$ is constant and independent of θ in accord with equation (1) for axial distances greater than approximately 800 mm or 60 equivalent diameters.

A more detailed account of the peripheral wall

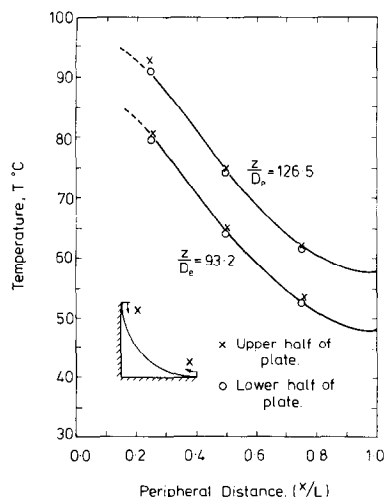


FIG. 3. Peripheral wall temperature variation at two axial locations.

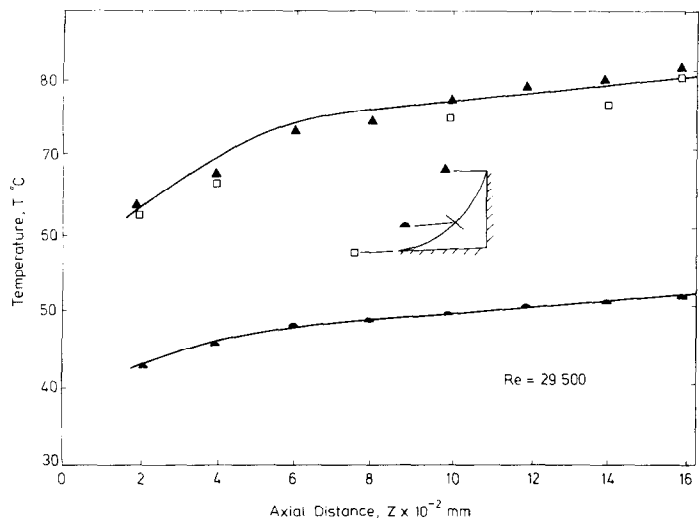


FIG. 4. Wall temperature distribution along the duct.

temperature distribution is presented in Fig. 5 where local wall temperatures are shown for a range of Reynolds numbers. These profiles refer to a constant energy input corresponding to an average surface heat flux equal to 1340 W m^{-2} . Most noticeable is the behaviour at small values of Reynolds number where the average wall temperature is large. In the idealized model, the maximum temperature must occur at the insulated corner of the duct: however, because of heat conduction into the wooden plane walls of the actual duct, the maximum wall temperature occurs at intermediate points as shown in Fig. 5. This is noticeable particularly at the largest level of wall temperature, i.e. when $Re = 5315$.

All the experimental temperature data presented so far refer to actual values. As has been indicated in the Introduction, the peripheral or azimuthal temperature difference in the wall of the duct is of special interest. Figures 6 and 7 show these temperature differences. In Fig. 6, for example, the temperature difference around

the wall (i.e. the temperature in the wall relative to the minimum wall temperature) is shown plotted vs peripheral distance from the corner, O . The data is for $Re = 42\,700$, and as can be seen, the greatest difference between the experimental data and the prediction occurs in the region of the corner, O , where the local temperature is reduced by heat loss. Otherwise there is good agreement between theory and practice. In Fig. 7, a local temperature difference [i.e. the temperature at $(x/L) = 0.55$ minus the minimum temperature at $(x/L) = 1$], is plotted against Re . Here the largest disagreement with the numerical prediction reflects what is observed in Fig. 5 concerning the heat conduction loss at small values of Reynolds number.

Finally, the fully developed average Nusselt numbers were calculated and these are compared with typical heat transfer correlations in Fig. 8. The most familiar correlation is the Dittus-Boelter equation

$$Nu = 0.023 Re^{0.8} Pr^{0.4}, \tag{2}$$

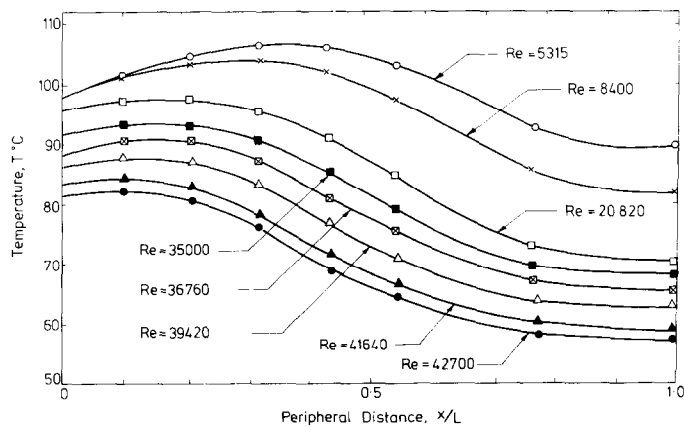


FIG. 5. Peripheral wall temperature variation at various Reynolds numbers ($z/D_e = 126.5$). (Average wall heat flux 1340 W m^{-2} .)

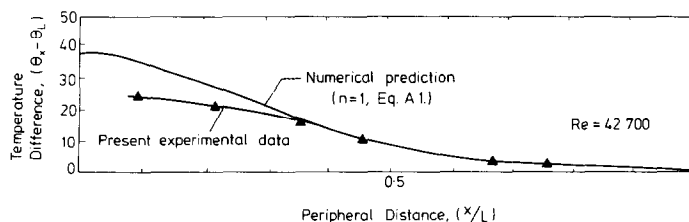


FIG. 6. Peripheral wall temperature variation.

while that due to Petukhov and Popov [7], is given by

$$Nu = \left(\frac{f}{8} \right) Re Pr / (1.07 + 12.7 (Pr^{2/3} - 1)(f/8)^{1/2}), \quad (3)$$

$$f = (1.82 \log_{10} Re - 1.64)^{-2}.$$

Following a recommendation proposed by Altemani and Sparrow [8], a modified Petukhov–Popov correlation has also been used. In this case the friction factor employed in equation (3) is the experimental value for the particular geometry in question. Therefore, the data shown in Fig. 2 has been used in the modified correlation plotted in Fig. 8. There is a marked difference between the present data for the average Nusselt numbers and the correlations. This again is probably due to the weakness of the equivalent diameter concept and the asymmetry of heating in the present experimental work. The narrow corners of the duct may produce regions of predominantly laminar flow which may persist to relatively large values of the Reynolds number resulting in the deterioration of the overall heat transfer as indicated. A critical examination of the trend of the experimental data in Fig. 8, however, reveals a gradual convergence towards the modified correlation. This is consistent with the reasoning of Gunn and Darling [6], for friction in noncircular ducts. That is, as the value of Re increases, it is to be expected that better agreement with modified correlation will be obtained using the equivalent diameter idea.

The two effects cannot be separated in heat transfer. Both, however, lead to smaller values of the Nusselt number so that the effects are additive. The

contribution of heat asymmetry to the difference between the experimental data and the correlation in Fig. 8, must be relatively small in the case of airflow, therefore the lack of agreement between the data and the correlation must be principally due to the effect of duct shape, i.e. inadequacy of the hydraulic radius concept.

These points concerning the average value of the Nusselt number are not relevant as far as the comparisons made in Figs. 3–7 are concerned, since the actual measured values were employed in the predictions.

CONCLUDING REMARKS

Experimental measurements of the peripheral temperature variation in the wall of a very noncircular duct have been made and compared with values predicted using a simple model for the conjugate wall heat conduction and fluid convection heat transfer. The differences between theory and experiment are a maximum at large wall temperatures and are attributable to heat losses, which are not included in this model, otherwise the results as a whole are very encouraging.

Evaluation of the friction factor and the average Nusselt number indicates that the use of the hydraulic radius is inadequate if turbulent pipe flow correlations are to be employed for the prediction of these parameters in very noncircular ducts. Some asymmetry of heat transfer in the experiment also contributes to the shortcomings of the accepted procedure of using the equations for pipe flow for other shaped ducts.

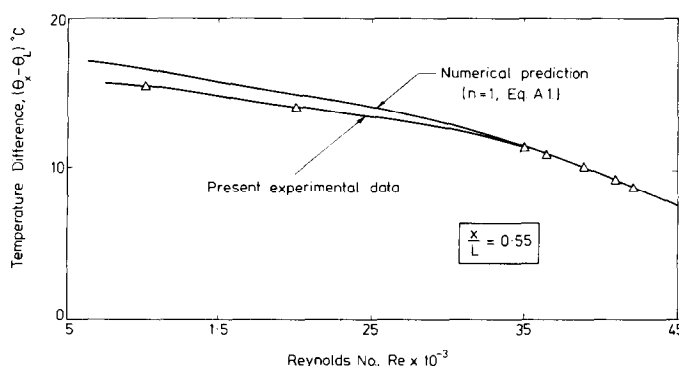


FIG. 7. Local peripheral temperature difference variation with Reynolds number.

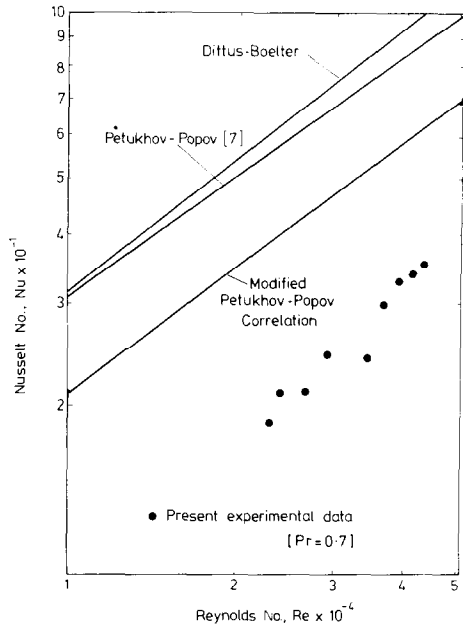


FIG. 8. Fully developed Nusselt numbers.

The work described is particularly relevant to the prediction of azimuthal temperature variation in the cladding of fuel elements of a pressurized water reactor in the event of a LOCA (loss of coolant accident). The complete numerical analysis of this problem is fraught with uncertainties and difficulties in execution, whereas the present elementary analysis requires relatively simple input data in the form of the wall parameters, an average convection heat transfer coefficient and an exponent to approximate the distribution of the local values of heat transfer coefficient around the boundary. It is considered that the present simple analysis as outlined in the Appendix, achieves much better accuracy than the complete numerical analysis of the coupled heat transfer problem, particularly in the case of turbulent flow.

With regard to future work and development, experiments covering a wide range of the duct wall thickness would be very useful. (Changes in thermal conductivity, particularly in the cladding problem, are only relatively small.) On the theoretical side, other functional relationships for the distribution of the heat transfer coefficient around the wall could be examined with minimal effort. In this connection, a simple linear relationship appears to be adequate in a preliminary assessment of wall temperature variation.

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APPENDIX

THE NUMERICAL PREDICTION OF THE PERIPHERAL WALL TEMPERATURE VARIATION

The heat transfer model for the prediction of the temperature in the peripheral or azimuthal direction in the wall of a noncircular duct is shown in Fig. A1. The wall, which is of uniform thickness, t , is subjected to uniform heating, q_w , on one face and variable convective heat transfer on the other face in the peripheral direction. The ends of the wall, which corresponds to part OA of the curved boundary, are insulated. Here O refers to the corner of the duct, while A is the central point about which the peripheral wall temperature profile is symmetrical.

The local convective heat transfer coefficient, h , is of course constant in the flow direction but increases around the wall from zero value at O and in real geometries is well approximated by

$$h = \bar{h}(n+1)\left(\frac{x}{L}\right)^n, \tag{A1}$$

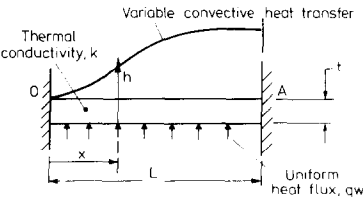
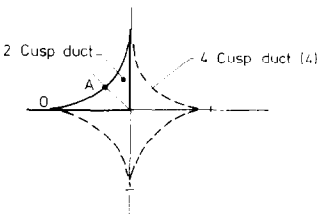


FIG. A1. Heat transfer model for the prediction of peripheral wall temperature distribution in a noncircular duct: (a) duct geometry; (b) heat transfer model for wall element OA.

where \bar{h} is an average value, which may be obtained either by direct experiment, as is the case here, or by an appropriate correlation for the particular geometry and conditions. In the present investigation actual experimental values for \bar{h} are used since they refer to the particular case in question and accommodate the effects of the asymmetry of heat transfer and shortcomings of the hydraulic radius concept in predicting the heat transfer rate. The value of n is chosen on the basis of data from other noncircular geometries, e.g. square and triangular ducts. If $n = 1$, for example, then the distribution of h is linear which is the simplest approximation. Likewise, if $n = 0$, then the situation is one where the local heat transfer coefficient is uniform as in flow in a pipe, or between parallel walls.

The theoretical analysis of heat transfer in the wall closely resembles that for a fin and the appropriate equation in the notation shown in Fig. (A1) is

$$\frac{d^2\theta}{dx^2} - \frac{h\theta}{kt} + \frac{q_w}{kt} = 0 \quad \{\theta = (T_w - T_b)\}, \quad (A2)$$

which, with equation (A1), becomes

$$\frac{d^2\theta}{dx^2} - \frac{\bar{h}(n+1)}{kt} \left(\frac{x}{L}\right)^n \theta + \frac{q_w}{kt} = 0. \quad (A3)$$

In order to effect a solution of equation (A3), it is first cast into finite-difference form following a procedure similar to that adopted by Gosman *et al.* [9]. Most conveniently, T_b is chosen to be zero, since the peripheral wall temperature profile is the same at all axial locations according to equation (1), and it is the magnitude of the temperature variation around the wall which is of interest. In the computation, the thermal conductivity k , wall thickness t , and heat flux q_w , are prescribed. The value of \bar{h} is obtained from experiment in this investigation since it is the prediction procedure which is to be tested here. Various values of n may be chosen to determine which one affords the best prediction for the particular geometry studied. It was found that $n = 1$ was satisfactory for present purposes. Typical results of the numerical solution are given in Figs. 6 and 7 and are discussed in Section 3.

VARIATION PERIPHERIQUE DE TEMPERATURE DANS LA PAROI D'UNE CONDUITE NON CIRCULAIRE; UNE ETUDE EXPERIMENTALE

Résumé—On considère la variation périphérique de la température dans la paroi d'une conduite droite et de section non circulaire, avec une référence spéciale au calcul de la température dans l'enveloppe des barres combustibles d'un réacteur à eau pressurisée, dans le cas d'une perte de réfrigérant. Un modèle simple de transferts thermiques conjugués dans la paroi et dans le fluide est utilisé pour calculer la variation de température autour de la paroi. Pour tester la théorie, des expériences ont été faites pour déterminer les températures de la paroi dans une conduite avec de l'air comme fluide à des débits variés. Des mesures de perte de charge et de transfert thermique pour obtenir le coefficient moyen de perte de charge et celui de convection, indiquent que le concept de rayon hydraulique est inadéquat dans le cas des sections non circulaires utilisées dans l'expérience et elles précisent l'effet de l'asymétrie dans le transfert thermique. On pense que ces expériences sont les premières dans ce genre.

PERIPHERER TEMPERATURVERLAUF IN DER WAND EINES NICHTKREISFÖRMIGEN KANALS—EINE EXPERIMENTELLE UNTERSUCHUNG

Zusammenfassung—Diese Arbeit befaßt sich mit dem peripheren Temperaturverlauf in der Wand eines geraden, nichtkreisförmigen Kanals. Besonders berücksichtigt wird dabei die Berechnung der Temperatur in den Hüllrohren der Brennstäbe eines Druckwasser-Reaktors im Fall eines Kühlmittelverlustes. Zur Berechnung des Temperaturverlaufs in Umfangsrichtung der Wand wird ein einfaches Modell für den gekoppelten Wärmetransport in Wand und Fluid benutzt. Zur Überprüfung der Theorie wurden Experimente zur Bestimmung der Wandtemperaturen in einem spitz auslaufenden Kanal durchgeführt, wobei Luft als Arbeitsmittel in einem Durchsatzbereich verwendet wurde. Gesamtdruckabfall- und Wärmeübergangsmessungen zur Bestimmung des Reibungsfaktors und des mittleren Wärmeübergangskoeffizienten zeigen die Unzulänglichkeit des Konzepts des hydraulischen Radius für den Fall der in den Experimenten vorliegenden, ausgeprägt nichtkreisförmigen Geometrie und auch den Einfluß der Asymmetrie im Wärmeübergang. Es wird angenommen, daß diese Wärmeübertragungs-Experimente die ersten für diese spezielle Geometrie sind.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ИЗМЕНЕНИЯ ТЕМПЕРАТУРЫ В СТЕНКЕ НЕКОЛЬЦЕВОГО КАНАЛА

Аннотация—Исследуется изменение температуры в стенке прямого не кольцевого канала. Особое внимание уделено расчету температуры в оболочке топливных стержней водяного реактора под давлением в случае утечки теплоносителя. Используется простая модель сопряженных процессов передачи тепла в стенке и жидкости для расчета изменений температуры по окружности стенки. Для проверки теории проведены эксперименты по определению температуры стенки суживающегося канала с использованием в качестве рабочей жидкости воздуха при различных скоростях течения. Измеренные значения суммарного перепада давления и величины теплового потока с учетом коэффициента трения и среднего коэффициента переноса тепла свидетельствуют о неадекватности использования понятия гидравлического радиуса для исследуемой не кольцевой геометрии и о влиянии асимметрии на теплоперенос. Считается, что подобные эксперименты для данной геометрии были выполнены впервые.